MATHEMATICS

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AIM POINTS
MATHEMATICS
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DIRECTION COSINES AND DIRECTION RATIOS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. If P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are two projection space, then

   PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

2. The distance of a point P (x, y, z) from the O is given by

   OP = $\sqrt{x^2 + y^2 + z^2}$

3. If P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are two point, then the coordinates of a point dividing PQ internally in the ratio m : n are

   $$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

   If R rivides PQ externally in the ratio m : n, then its coordinates are

   $$\left(\frac{mx_2 + nx_1}{m-n}, \frac{my_2 + ny_1}{m-n}, \frac{mz_2 + nz_1}{m-n}\right)$$

4. The line segment joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is divided by
   (i) YZ-plane in the ratio $-x_1 : x_2$
   (ii) ZX-plane in the ratio $-y_1 : y_2$
   (iii) XY-plane in the ratio $-z_1 : z_2$

5. The coordinates of the centroid of the triangle formed by the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

   $$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

6. The coordinates of the centroid of the triangle formed by the points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), (x_4, y_4, z_4) is

   $$\left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4}\right)$$

7. The distance of point P (x, y, z) from x, y and z axes are $\sqrt{y^2 + z^2}$, $\sqrt{z^2 + x^2}$ and $\sqrt{x^2 + y^2}$ respectively.

8. If a directed line segment OP makes anglea α, β, γ with OX, OY and OZ respectively, then cos α, cos β, cos γ are known as the direction cosines of OP and are generally denoted by l, m, n.
   Thus, we have $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$
   Direction cosines of PO are $-l, -m, -n$.
   If OP = r and the coordinates of P are (x, y, z), then $x = lr$, $y = mr$, $z = nr$.

9. If l, m, n are direction cosines of a vector $\vec{r}$. then
   (i) $\vec{r} = |\vec{r}|\,(l\hat{i} + m\hat{j} + n\hat{k}) \Rightarrow \vec{r} = l\hat{i} + m\hat{j} + n\hat{k}$
   (ii) $l^2 + m^2 + n^2 = 1$

(iii) Projection of $\vec{r}$ on the coordinates axes are $l\,|\vec{r}|, |\vec{r}|\,m, |\vec{r}|\,n$

(iv) $|\vec{r}| = \sqrt{\text{Sum of the squares of projections of } \vec{r} \text{ on the coordinate axes}}$

10. If P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are two points such that the direction cosines of $\overrightarrow{PQ}$ are l, m, n. Then,

$x_2 - x_1 = l\,|\overrightarrow{PQ}|, y_2 - y_1 = m\,|\overrightarrow{PQ}|, z_2 - z_1 = n\,|\overrightarrow{PQ}|$

These are projection of $\overrightarrow{PQ}$ on X, Y and Z-axes respectively.

11. If l, m, n are direction cosines of a vector and a, b, c are three numbers such that $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

Then, we say that the direction ratios of $\vec{r}$ are proportional to a, b, c.
Also, we have

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

12. If θ is the angle between two lines having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2, then $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

(i) Lines are perpendicular, iff $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(ii) Lines are perpendicular, iff $l_1l_2 + m_1m_2 + n_1n_2 = 0$

13. IIf θ is the angle between two lines whose direction ratios are proportional to a_1, b_1, c_1 and a_2, b_2, c_2 respectively, then the angle θ between them is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Lines are parallel, iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Lines are perpendicular, iff $a_1\,a_2 + b_1\,b_2 + c_1\,c_2 = 0$

14. The projection of the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) to the line having direction cosines l, m, n is

$|(x_2 - x_1)\,l + (y_2 - y_1)\,m + (z_2 - z_1)\,n|$

15. The direction ratios of the line passing through points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are proportional to $x_2 - x_1, y_2 - y_1, z_2 - z_1$

Direction cosines of $\overrightarrow{PQ}$ are

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$$

EXERCISE-1

1. Prove that the distance between the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by

   $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

2. Let P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) be two point. Let R be a point on the line segment joining P and Q such that it divides the join of P and Q internally in the ratio $m_1 : m_2$. Then, the coordinates of R are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$

3. Find the coordinates of the point which divides the join of P(2, –1, 4) and Q(4, 3, 2) in the ratio 2 : 3 (i) internally (ii) externally.

4. Find the ratio in which the line joining the points (1, 2, 3) and (–3, 4, –5) is divided by the xy-plane. Also find the coordinates of the point of division.

5. Using section formula, prove that the three point A(–2, 3, 5), B(1, 2, 3) and C(7, 0, –1) are col-linear.

6. Find the coordinates of the points which trisect the line segment AB, given that A(2, 1, –3), and B(5, –8, 3).

7. Show that the centroid of the triangle with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

8. Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4).

9. Find the angle between the lines whose direction ratios are proportional to 4, –3, 5 and 3, 4, 5.

10. Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining B(1, 4, 6) and C(5, 4, 4).

11. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two lines, show that the direction cosines of the line perpendicular to both of them are proportional to $(m_1n_2 - m_2n_1), (n_1l_2 - n_2l_1), (l_1m_2 - l_2m_1)$.

12. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two mutually perpendicular lines, show that the direction consies of the line perpendicular to both of them are $(m_1n_2 - m_2n_1), (n_1l_2 - n_2l_1), (l_1m_2 - l_2m_1)$.

13. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, –4), (–1, 1, 2) and (–5, –5, –2) and also find the angles of the triangle of the triangle. What types of triangle it is ?

14. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$, $6mn - 2nl + 5lm = 0$.

15. Prove that the straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $f\,mn + g\,nl + h\,lm = 0$ are perpendicular, if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and paralel, if $a^2f^2 + b^2g^2 + c^2h^2 - 2abfg - 2bcgh - 2achf = 0$

16. If the edges of a rectangular parallelopiped are a, b, c; prove that the angles between the four diagonal are given by

$$\cos^{-1}\left(\frac{a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right).$$

17. Show that the angles between the diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

EXERCISE-2

1. Find the angle between the vectors with direction ratios 1, –2, 1 and 4, 3, 2.
2. Find the angle between the vectors whose direction cosines are proportional to 2, 3, –6 and 3, –4, 5.
3. Find the direction cosines of the lines, connected by the relations : $l + m + n = 0$ and $2lm + 2ln - mn = 0$.
4. Find the angle between the lines whose direction cosines are given by the equations
   (i) $l + m + n = 0$ and $l^2 + m^2 - n^2 - 0$
   (ii) $2l - m + 2n = 0$ and $mn + nl = lm = 0$
   (iii) $l + 2m + 3n = 0$ and $3lm - 4ln + mn = 0$
5. Find the acute angle between the lines whose direction ratios are 2 : 3 : 6 and 1 : 2 : 2.
6. Find the angle between the lines whose direction ratios are proportional to a, b, c and b – c, c – a, a – b.
7. If the coordinates of the points A, B, C, D are (1, 2, 3), (4, 5, 7), (–4, 3, –6) and (2, 9, 2), then find the angle between AB and CD.

EXERCISE-3

1. A line makes an angle of 60° with each of X-axis and y-axis. Find the acute angle made by the line with Z-axis.
2. If a line makes angles α, β and γ with the coordinate axes, find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.
3. Write the ratio in which the line segment joining (a, b, c) and (–a, –c, –b) is divided by the xy-plane.
4. Write the coordinates of the projection of point (x, y, z) on XOZ-plane.
5. Find the distance of the point (2, 3, 4) from the x-axis.